South East Asian J. of Mathematics and Mathematical Sciences
Vol. 21, No. 2 (2025), pp. 161-174

DOI: 10.56827/SEAJMMS.2025.2102.12 **ISSN (Online): 2582-0850**

ISSN (Print): 0972-7752

EXPLORING SEPARATION AXIOMS IN BINARY MULTISET

S. P. R. Priyalatha and R. Sowndariya

Department of Mathematics,
Kongunadu Arts and Science College,
Coimbatore - 641 029, Tamil Nadu, INDIA

E-mail : priyalathamax@gmail.com, sowndariyainf@gmail.com

(**Received:** *Oct. 24, 2024* **Accepted:** *Aug. 05, 2025* **Published:** *Aug. 30, 2025*)

Abstract: In this paper, the concept of separation axioms in binary multiset topological spaces is explored. The separation axioms are introduced and some of their properties are investigated. Furthermore, the properties of being T_0, T_1, T_2, T_3, T_4, T_5, and $T_{2\frac{1}{2}}$ spaces are proved to be hereditary.

Keywords and Phrases: bms T_0-space, bms T_1-space, bms T_2-space, bms T_3-space, bms T_4-space, bms T_5-space.

2020 Mathematics Subject Classification: 00A05, 03E70, 54B10, 54G20.

1. Introduction

The field of topology serves as a cornerstone of modern mathematical analysis, providing insights into the properties and relationships of space that are preserved under continuous transformations. A central theme within this discipline is the study of separation axioms, which delineate the conditions under which distinct points and sets can be distinguished from one another within a topological space. Separation axioms, traditionally represented as T_0, T_1, T_2, T_3, and T_4 form a critical framework for understanding the topology of various spaces. These axioms not only facilitate a classification of spaces but also underpin many fundamental theorems and concepts in topology. Recent contributions by researchers such as Tong [13], who analyzed the relationships between T_0 and T_1 spaces. The basic properties of the multiset can be found in Girish [5], who explored multisets in relation to topological properties, have enriched this discourse. Additionally, Hoque, et.al.,

[6] introduction of the multiset separation axiom and Nithyanantha Jothi [7, 8, 9] development of binary topological spaces highlight the evolving nature of this field and separation axioms of various condition in binary multiset topological space. Sobhy , et.al. [4] portrayed the separation axioms in multiset topological space established the T_0, T_1, T_2, T_3, T_4, and T_5 and theorem and properties are derived. Building on these advancements, Amer [1] innovative work on binary D-separation axioms offers fresh perspectives on the relationships among derived points and kernels within binary topological spaces. Sureka and Sindhu [11] provided the new class of open binary multiset topological space and their overview of properties and also Sharavan [12] established in metrizability of multiset topological space are defined theorem and examples. Priyalatha, et.al., [10] introduced new topological for the binary multiset topological space and presented some theorem and examples. Also, the multiset topological space established the application of the genetic mutation of DNA and RNA for the basic properties and examples are presented [2] [3] {G, G, G, T, T, T, T, A, C, C, A, A} has an equivalent representation in bms {3/G, 4/T, 2/C, 3/A }.
This work is organized as follows: After this introduction, in Section 2, recall some definitions and results that are required to make this work self-contained. In Section 3, introduce the concepts of functionally T_i spaces for $i = 0, 1, 2, 3, 4, 5, T_{2\frac{1}{2}}$, and study their definitions and theorems. In Section 4, examine their main properties, especially those related to bms topological spaces. In Section 5, study the characterization of the bms separation axioms and their properties. Finally conclude with some remarks and propose future work in Section 6.

2. Basic Preliminaries

Definition 2.1. [7] *A binary topological space* (X, Y, M) *is called a binary* $-T0$ *if for any two binary points* $(x_1, y_1), (x_2, y_2) \in X \times Y$ *with* $x_1 \neq x_2$, $y_1 \neq y_2$, *there exists* $(A, B) \in M$ *such that exactly one of the following holds:*

1. $(x_1, y_1) \in (A, B)$, $(x_2, y_2) \in (X \setminus A, Y \setminus B)$, *and*
2. $(x_1, y_1) \in (X \setminus A, Y \setminus B)$, $(x_2, y_2) \in (A, B)$.

Definition 2.2. [7] *A binary topological space* (X, Y, M) *is called a binary-T1 if for every* $(x_1, y_1), (x_2, y_2) \in X \times Y$ *with* $x_1 \neq x_2$, $y_1 \neq y_2$, *there exist* $(A, B), (C, D) \in M$, *with* $(x_1, y_1) \in (A, B)$ *and* $(x_2, y_2) \in (C, D)$ *such that* $(x_2, y_2) \in (X \setminus A, Y \setminus B)$ *and* $(x_1, y_1) \in (X \setminus C, Y \setminus D)$.

Definition 2.3. [9] *The binary points* $(x_1, y_1), (x_2, y_2) \in X \times Y$ *are distinct if* $x_1 \neq x_2$, $y_1 \neq y_2$.

Definition 2.4. [9] *The ordered pair* $((A, B)^\circ_1, (A, B)^\circ_2)$ *is called the binary interior of* (A, B)*, denoted by b-int*(A, B).

Definition 2.5. [9] *The ordered pair* $((A, B)^*_1, (A, B)^*_2)$ *is called the binary closure of* (A, B)*, denoted by b-cl*(A, B) *in the binary space* (X, Y, M) *where* $(A, B) \subseteq (X, Y)$.

Definition 2.6. [9] *Let* (X, Y, M) *be a binary topological space and let* $(x, y) \in X \times Y$*. The binary open set* (A, B) *is called a binary neighborhood of* (x, y) *if* $x \in A$ *and* $y \in B$.

Definition 2.7. [1] *Let* $M \in [X]^w$ *and* $\tau \subseteq P^*(M)$*. Then,* τ *is called a multiset topological space of M if* τ *satisfies the following properties.*
(i) The mset M and the empty mset ϕ *are in* τ.
(ii) The mset union of the elements of any sub collection of τ *is* τ.
(iii) The mset Intersection of the elements of any finite sub collection of τ *is in*τ.

Definition 2.8. [1] *Given a submset A of an M-topological space M in* $[X]^w$*, the interior of A is defined as the mset union of all open mset contained in A and its denoted by* $Int(A)$*. i.e., Int(A)=*$\cup\{G \subseteq M : G$ *is an open mset and* $G \subseteq A\}$ *and* $C_{Int(A)}(x) = max\{C_G(x) : G \subseteq A\}$.

Definition 2.9. [1] *Given a submset A of an M-topological space space M in* $[X]^w$*, the closure of A is defined as the mset intersection of all closed mset containing A and its denoted by Cl*(A)*. i.e., Cl*$(A) = \cap\{K \subseteq M : K$ *is a closed mset and* $A \subseteq K\}$ *and* $C_{Cl(A)}(x) = min\{C_K(x) : A \subseteq K\}$.

Definition 2.10. [9] *Let X And Y be any two no empty sets. A binary topological space X to Y is a binary structure* $M \subseteq \rho(X) \times \rho(Y)$ *that satisfies the following axioms.*
(i) (ϕ, ϕ) *and*$(X, Y) \in M$.
(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in M$ *whenever* $(A_1, B_1) \in M, (A_2, B_2) \in M$.
(iii) If $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$ *is a family of members of M, Then,* $(\bigcup_\alpha \in \Delta A_\alpha, \bigcup_\alpha \in \Delta B_\alpha) \in M$.

Definition 2.11. [2] *In biology, mutations are changing to the nucleotide sequence of the genetic material of an organism. Mutations might happen by copying errors in genetic material during cell division. Classification of mutations is based on the following: Effect on structure, Effect on function, Aspect on phenotype affected, Effect on inheritance. Structurally, mutations can be classified as follows: small-scale mutations (affecting small genes in one or few nucleotides), large-scale mutations (affect chromosomal structure). Small scale: One gene is affected by any change to*

the DNA sequence of a gene: nucleotides/bases may be added, missed, or changed: point mutation, insertions, deletions.
1. Point mutations: These occur within the protein-coding region of a gene which is classified into the following three types according to the erroneous codon codes:
(i) Silent mutation through which there is a code for the same amino acid.
(ii) Missense mutation through which there is code for different amino acids.
(iii) Nonsense mutation through which there is a code for stop and can truncate the protein.
(iv) Netural mutation through which there is a detectable change in the function of the protein.
2. Frameshift mutation: It is usually caused by errors during replications of repeating elements; insertions in the coding region of a gene cause; a shift in the reading frame (frameshift); one or more bases (A, T, C, or G) are added or deleted. A lot of researchers use biological methods for the identification and repair of mutations.

Note 2.12. [10] *We know that, the power set of a m-set $M_1(resp.M_2)$ is the support set of the power m-set of $M_1(resp.M_2)$, is symbolized by $P^*(M_1)(resp.P^*(M_2))$. We can define $P^*(M_1) \times P^*(M_2) = \{(A_i, B_i) : A_i \in P^*(M_1), B_i \in P^*(M_2)\}$. According this definition, the ordered pair $(\{A,B\})$ is called a bms from M_1 and M_2 where $A \subseteq M_1$ and $B \subseteq M_2$. That is, the bms $(\{A,B\})$ is an element in $P^*(M_1) \times P^*(M_2)$.*

Definition 2.13. [10] *Let $M_1 \in [U]^w$, $M_2 \in [V]^r$ be two m-sets drawn from U and V respectively. A binary multiset topology (briefly, bms-topology) from M_1 to M_2 is a binary multiset structure $\tau_b \subseteq P^*(M_1) \times P^*(M_2)$ that satisfies the following axioms:*

(i) $(\phi, \phi), (M_1, M_2) \in \tau_b$

(ii) *If* $(\{A_1, B_1\}), (A_2, B_2) \in \tau_b$, *then* $(A_1 \cap A_2), (B_1 \cap B_2) \in \tau_b$.

(iii) *If* $\{(A_\lambda, B_\lambda) : \lambda \in J\} \subseteq \tau_b$, *then* $(\cup A_\lambda, \cup B_\lambda) \in \tau_b$.
In this case, the structure (M_1, M_2, τ_b) is called bms-topological space(or bms-space).
Note that τ_b is an ordinary set whose elements are bms.

Definition 2.14. [10] *For a bms-space (M_1, M_2, τ_b), we have*

(i) Each element in τ_b is called an open binary multiset (or open bms) and the complement of open bms is named a closed binary multiset (or closed bms).

(ii) A sub-bms $(\{A,B\})$ of a bms-space (M_1, M_2, τ_b) is said to be closed bms if $(\{A, B\})^c = (M_1 \ominus A, M_2 \ominus B)$ is an open bms.

Definition 2.15. [10] *Let* (M_1, M_2, τ_b) *be an bms-topological space and* $A \subseteq M_1$, $B \subseteq M_2$. *Then* $(\{A,B\})$ *is closed bms in* (M_1, M_2, τ_b) *if* $(M_1 \ominus A, M_2 \ominus B) \in \tau_b$, *the complement of closed bms* τ_b^c.

Definition 2.16. [10] *The ordered pair* $(\{A_1, B_1\})^*, (A_2, B_2)^*$ *is called bms closure of* $(\{A,B\})$ *is defined as the intersection of all closed bms containing in* $(\{A,B\})$ *denoted by* $cl_b(\{A, B\})$ *is bms topolgical space* (M_1, M_2, τ_b) *where* $(\{A, B\}) \subseteq (M_1, M_2)$, $cl_b(\{A, B\}) or (\{\overline{A}, \overline{B}\}) = \cap\{(\{G, H\}) \subseteq (M_1, M_2) : (\{G, H\})$ *is a closed bms and* $(\{A,B\}) \subseteq (\{ G,H\})\}$.

Definition 2.17. [10] *A non-empty bms* (A, B) *is called a sub-bms of a BMS* (M_1, M_2) *if each element of* (A, B) *is in* (M_1, M_2), *symbolically:* $(A, B) \subseteq (M_1, M_2)$ *if and only if* $A \subseteq M_1$ *and* $B \subseteq M_2$. *In this case,* (M_1, M_2) *is called a super bms of* (A, B).

Definition 2.18. [10] *Let* (M_1, M_2, τ_b) *be a bms-space,* $(\{k/u\}, \{m/v\}) \in (M_1, M_2)$, *and* $(G, H) \subseteq (M_1, M_2)$. *Then,* (G, H) *is called a bms neighborhood (or bms-nbd) of* $(\{k/u\}, \{m/v\})$ *if there exists an open BMS* $(P, Q) \in \tau_b$ *such that:* $(\{k/u\}, \{m/v\}) \in (P, Q) \subseteq (G, H)$.

3. Separation Axioms in Binary Multiset Topological Space

In this section, introduce the binary multiset in separation axioms of T_i for i=0,1,2,3,4,5, and $T_{2\frac{1}{2}}$ their basic definitions, theorems and examples.

Definition 3.1. *A bms topological space* (M_1, M_2, τ_b) *is said to be bms* T_0*- space. If for any two jointly distinct points* $(\{p_1/x_1, q_1/y_1\})$ *and* $(\{p_2/x_2, q_2/y_2\}) \subseteq (M_1, M_2)$, *there exist* $(A, B) \in \tau_b$ *such that either* $(\{p_1/x_1, q_1/y_1\}) \subseteq (A, B)$ *and* $(\{p_2/x_2, q_2/y_2\}) \nsubseteq (M_1/A, M_2/B)$, *or* $(\{p_1/x_1, q_1/y_1\}) \nsubseteq (M_1/A, M_2/B)$ *and* $(\{p_2/x_2, q_2/y_2\}) \subseteq (A, B)$.

Example 3.2. Let $M_1 = \{1/a, 2/b, 3/c\}, M_2 = \{1/d, 3/e, 2/f\}$ be a multiset and $\tau_b = \{(\emptyset, \emptyset), (M_1, M_2), (\{1/a\}, \{3/e\}), (\{1/b, 1/c\}, \{2/f, 2/e\}), (\{1/d\}, \emptyset)\}$. Its clear that , (M_1, M_2, τ_b) is bms T_0-space and $(U, V) = (\{1/b, 1/c\}, \{2/f, 2/e\}) \subseteq \tau_b$.

Theorem 3.3. *The bms* T_0 *property is hereditary.*

Proof. Let (M_1, M_2, τ_b) be a bms T_0-space, and $(A, B) \subseteq M_1, (C, D) \subseteq M_2$. Then $\tau_b = \{(A, B) \cap (C, D) : (M_1, M_2) \in \tau_b\}$. For any distinct points $(\{p_1/x_1, q_1/y_1\}) \in (A, B)$ and $(\{p_2/x_2, q_2/y_2\}) \in (C, D)$ and $(\{p_2/x_2, q_2/y_2\}) \notin (A, B)$, or $(\{p_1/x_1, q_1/y_1\}) \notin (C, D)$ and $(\{p_2/x_2, q_2/y_2\}) \in (C, D)$. Since, $\tau_b = \{(A, B) \cap (C, D) : (M_1, M_2) \in \tau_b\}$, $M_1 \cap (A, B)$ and $M_2 \cap (C, D)$ as the disjoint open bms in τ_b $(\{p_1/x_1, q_1/y_1\})$ and $(\{p_2/x_2, q_2/y_2\})$. Hence, $(\{p_1/x_1, q_1/y_1\}) \in M_1 \cap (A, B)$ and $(\{p_2/x_2, q_2/y_2\}) \notin M_2 \cap (C, D)$, or $(\{p_1/x_1, q_1/y_1\}) \notin M_1 \cap (A, B)$ and $(\{p_2/x_2, q_2/y_2\})$

$\in M_2 \cap (C, D)$.

Definition 3.4. *Let (M_1, M_2, τ_b) be a bms topological space. If for every order pair of the bms $(\{p_1/x_1, q_1/y_1\})$, $(\{p_2/x_2, q_2/y_2\}) \subseteq (M_1, M_2)$, and $(\{p_1/x_1, q_1/y_1\}) \neq (\{p_2/x_2, q_2/y_2\})$, $(A, B), (C, D) \in \tau_b$, there exist disjoint points $(\{p_1/x_1,\ q_1/y_1\}) \subseteq (A, B), (\{p_2/x_2, q_2/y_2\}) \nsubseteq (A, B)$ and $(\{p_1/x_1, q_1/y_1\}) \nsubseteq (C, D), (\{p_2/x_2, q_2/y_2\}) \subseteq (C, D)$. It is known as (M_1, M_2, τ_b) is bms T_1-space.*

Example 3.5. Let $M_1 = \{1/f, 3/g\}, M_2 = \{4/p, 7/q\}$ be a bms and $\tau_b = \{(\emptyset, \emptyset), (M_1, M_2), (\{1/f\}, \{5/q\}), (\{2/g\}, \{2/p\}), (\{1/f\}, \emptyset)\}$. It's clear that bms T_0-space, but not bmsT_1- space, becasue there exist $(\{1/f\}, \{5/q\}) \subseteq \tau_b$, such that $(\{1/f\}) \neq (\{5/q\})$.

Theorem 3.6. *The property of being bms T_1- space is hereditary property.*
Proof. By the proof based on theorem 3.3.

Theorem 3.7. *Let (M_1, M_2, τ_b) be a bms topological space. If $(\{P/X\}, \{Q/Y\})$ is τ_b closed bms $\forall(\{p_1/x_1, q_1/y_1\}), (\{p_2/x_2, q_2/y_2\}) \in (M_1, M_2)^c$.*
Proof. Let $(\{P/X\}, \{Q/Y\}) \subseteq (M_1, M_2)$, such that $(\{p_1/x_1, q_1/y_1\}) \neq (\{p_2/x_2, q_2/y_2\})$, by hypothesis $(\{p_1/x_1, q_1/y_1\}), (\{p_2/x_2, q_2/y_2\})$ are τ_b-closed bms on (M_1, M_2). Then, $(\{p_1/x_1, q_1/y_1\})^c, (\{p_2/x_2, q_2/y_2\})^c \in \tau_b$. Since, $(\{P/X\}) \nsubseteq (\{p_2/x_2, q_2/y_2\})^c, (\{Q/Y\}) \subseteq (\{p_2/x_2, q_2/y_2\})^c$ and $(\{P/X\}) \subseteq (\{p_1/x_1, q_1/y_1\})^c, (\{Q/Y\}) \nsubseteq (\{p_1/x_1, q_1/y_1\})^c$. Hence (M_1, M_2, τ_b) is bms T_0-space.

Definition 3.8. *Let (M_1, M_2, τ_b) be a bms-topological space. If for every two pair of elements in bms $(\{p_1/x_1, q_1/y_1\})$ and $(\{p_2/x_2, q_2/y_2\}) \subseteq (M_1, M_2)$, $(p_1/x_1, q_1/y_1) \neq (\{p_2/x_2, q_2/y_2\})$, Then, $(A, B), (C, D) \in \tau_b$, there exist disjoint points $(\{p_1/x_1, q_1/y_1\}) \subseteq (A, B)$, $\{q_1/y_1, q_2/y_2\} \subseteq (C, D)$ and $(A, B) \cap (C, D) = \emptyset$. It is known as (M_1, M_2, τ_b) is bms T_2-space.*

Theorem 3.9. *Any discrete bms topology from M_1 to M_2 is bms T_0.*
Proof. Let (M_1, M_2, τ_b) be a discrete bms topological space and $(p_1/x_1, q_1/y_1)$, $(p_2/x_2, q_2/y_2) \in M_1 \times M_2$ with $(\{p_1/x_1\}) \neq (\{p_2/x_2\})$ or $(\{q_1/y_1\}) \neq (\{(q_2/y_2)\})$. Since, τ_b is a discrete bms topology from M_1 to M_2, $(\{p_1/x_1, q_1/y_1\}) \in \tau_b$. Therefore, $(\{p_1/x_1, q_1/y_1\}) \in (\{p_1/x_1, q_1/y_1\})$ and $(\{p_2/x_2, q_2/y_2\}) \in (M_1 \setminus \{p_1/x_1\}, M_2 \setminus (\{q_1/y_1\}))$ and $(\{p_1/x_1, q_1/y_1\}), (\{p_2/x_2, q_2/y_2\}) \in M_1 \times M_2$ with $(\{p_1/x_1\}) \neq (\{p_2/x_2\})$ or $(\{q_1/y_1\}) \neq (\{q_2/y_2\})$. Hence (M_1, M_2, τ_b) is bms T_0.

Theorem 3.10. *Every indiscrete bms topology (M_1, M_2, τ_b) is not bms T_2-space where (M_1, M_2) more than or equal two different bms-points.*
Proof. It follows that Theorem 3.9.

Theorem 3.11. *Every bms T_2-space is a bms T_1-space.*

Proof. Let (M_1, M_2, τ_b) be a bms T_2-space and $(\{p_1/x_1, q_1/y_1\}), (\{p_2/x_2, q_2/y_2\}) \subseteq (M_1, M_2)$ therefore $(\{p_1/x_1, q_1/y_1\}) \neq (\{p_2/x_2, q_2/y_2\})$, by the definition 3.8, $(M_1 \cap (A, B)) \cap (M_2 \cap (C, D)) = ((A, B) \cap (C, D)) \cap (M_1, M_2) = \emptyset \cap (M_1, M_2) = \emptyset$. Therefore, $(\{p_1/x_1, q_1/y_1\}) \subseteq (A, B)$, $(\{p_2/x_2, q_2/y_2\}) \subseteq (C, D)$. Then $(\{p_1/x_1, q_1/y_1\}) \nsubseteq (C, D)$, $(\{p_2/x_2, q_2/y_2\}) \nsubseteq (A, B)$.

Definition 3.12. *A bms topological space (M_1, M_2, τ_b) is bms - regular if and only if for every closed bms of sub-bms in $(A, B), (C, D)$ of (M_1, M_2) and each point $(\{p_1/x_1, q_1/y_1\}), (\{p_2/x_2, q_2/y_2\}) \in (M_1, M_2)$ there exist two disjoint open bms in $((\{p_1/x_1, q_1/y_1\})), (\{p_2/x_2, q_2/y_2\})$. Therefore $(A, B) \subseteq (C, D)$ and $(A, B) \cap (C, D) = \emptyset$.*

Definition 3.13. *Let (M_1, M_2, τ_b) be a bms-topoloigcal space. If for every disjoint points $(Q_1, Q_2) \in \tau_b^c$, and $((\{p_1/x_1, q_1/y_1\})), (\{p_2/x_2, q_2/y_2\}) \nsubseteq (Q_1, Q_2)$, there exist $(A, B), (C, D) \in \tau_b$, such that $(Q_1, Q_2) \subseteq (A, B), ((\{p_1/x_1, q_1/y_1\})) \subseteq (A, B), (\{q_1/y_1, q_2/y_2\}) \nsubseteq (C, D)$, and $(A, B) \cap (C, D) = \emptyset$. It is known as (M_1, M_2, τ_b) is bms-regular space.*

Definition 3.14. *A bms-topological space (M_1, M_2, τ_b) is said to be a bms T_3-Space if:*
1.(M_1, M_2, τ_b) is bms-regular space.
2. (M_1, M_2, τ_b) is bms T_1-space.

Note 3.15. *Every discrete bms topology from M_1 to M_2 is bms T_3-space.*

Theorem 3.16. *Let (M_1, M_2, τ_b) be a bms-regular space and $(A, B) \subseteq (M_1, M_2)$. then (A,B) is bms- regular space.*
Proof. Let sub-bms of (A,B) in bms topological space $(\{p_1/x_1, q_1/y_1\}) \subseteq M_1$, $(\{p_1/x_1, q_1/y_1\}) \nsubseteq M_2$, and τ_b-closed bms in (M_1, M_2), such that $(A, B) = (Q_1, Q_2) \cap V$. Since, $(\{p_1/x_1, q_1/y_1\}) \nsubseteq (A, B)$. Then, $(\{p_2/x_2, q_2/y_2\}) \nsubseteq (Q_1, Q_2)$, (M_1, M_2, τ_b) be bms-regular space, $(Q_1, Q_2) \subseteq (A, B), (\{p_2/x_2, q_2/y_2\}) \subseteq (C, D)$ and $(A, B) \cap (C, D) = \emptyset$. Then, $(Q_1, Q_2) \cap (U, V) \subseteq (A, B) \cap (U, V), (\{p_1/x_1, q_1/y_1\}) \subseteq (A, B) \cap (U, V)$, and $((A, B) \cap (U, V)) \cap ((C, D) \cap (U, V)) = ((A, B) \cap (C, D)) \cap (U, V) = \emptyset \cap (U, V) = \emptyset$. Hence (A, B) is bms-regular space.

Definition 3.17. *Let (M_1, M_2, τ_b) be a bms-topological space. If for every disjoint points $(Q_1, Q_2) \in \tau_b^c$, and $Q_1 \cap Q_2 = \emptyset$, Then, there exist $(E, F) \in \tau_b$, such that $Q_1 \subseteq E, Q_2 \subseteq F$, and $E \cap F = \emptyset$. Hence, (M_1, M_2, τ_b) is bms-normal space.*

Definition 3.18. *A bms-topological space (M_1, M_2, τ_b) is said to be a bms T_4-space if:*
1. (M_1, M_2, τ_b) is bms-normal space.

2. (M_1, M_2, τ_b) is bms T_1-space.

Theorem 3.19. *Every closed Sub-bms space of bms-normal space is also a bms-normal space.*

Definition 3.20. *Let (M_1, M_2, τ_b) be a bms-topological space and let(A,B) $\subseteq (M_1, M_2)$ is a two non-empty bms. Then, (A,B) are separated bms if $A \cap \overline{B} = \emptyset, \overline{A} \cap B = \emptyset$.*

Definition 3.21. *A bms-topological space (M_1, M_2, τ_b) is said to be bms-completely normal space iff for any two separated sub-bms (A,B) of (M_1, M_2) there exist $(E, F) \in \tau_b$ such that $(A, B) \subseteq (E, F)$.*

Theorem 3.22. *Every bms-completely normal space is bms-normal space.*

Proof. Let (M_1, M_2, τ_b) be a bms-completely normal space and (A,B) sub-bms of (M_1, M_2) such that, $A \cap B = \emptyset$ and $A \cap \overline{B} = \emptyset$. Therefore, $(E, F) \in \tau_b$, $(A, B) \subseteq (E, F)$ and $E \cap F = \emptyset$. Hence (M_1, M_2, τ_b) is a bms-normal space.

Definition 3.23. *Let (M_1, M_2, τ_b) be a bms-topological space. If for every two singleton $(\{p_1/x_1, q_1/y_1\}), (\{q_1/y_1, q_2, y_2\}) \subseteq (M_1, M_2)$ such that $(\{p_1/x_1, q_1/y_1\}) \neq (\{p_2/x_2, q_2/y_2\})$ and $(E, F) \in \tau_b$, $(\{p_1/x_1, q_1/y_1\}) \subseteq E$, $(\{q_1/y_1, q_2, y_2\}) \subseteq F$, $\overline{E} \cap \overline{F} = \emptyset$. Hence (M_1, M_2, τ_b) is bms $T_{2\frac{1}{2}}$-Space.*

Note 3.24. *Every discrete bms-topology $(M_1, M_2, P^*(M_1, M_2))$ is bms $T_{2\frac{1}{2}}$-Space.*

Theorem 3.25. *If (M_1, M_2, τ_{b_1}) is bms $T_{2\frac{1}{2}}$-space and $\tau_{b_1} \leq \tau_{b_2}$, Then, (N_1, N_2, τ_{b_2}) is also bms $T_{2\frac{1}{2}}$-space.*

Proof. Let (M_1, M_2, τ_{b_1}) be a bms $T_{2\frac{1}{2}}$- space and $\tau_{b_1} \leq \tau_{b_2}$. Every open bms in τ_{b_1} is also an open bms in τ_{b_2}. Since, (N_1, N_2, τ_{b_2}) is bms $T_{2\frac{1}{2}}$- space, $(\{p_1/x_1, q_1/y_1\})$ and $(\{p_2/x_2, q_2/y_2\})$ in (N_1, N_2), $(\{p_1/x_1, q_1/y_1\}) \neq (\{q_1/y_1, \underline{q_2/y_2}\})$, there exist $(E, F) \in \tau_{b_2}$, $(\{p_1/x_1, q_1/y_1\}) \subseteq E$, $(\{p_2/x_2, q_2/y_2\}) \subseteq F$, and $\overline{E} \cap \overline{F} = \emptyset$. Since, (M_1, M_2, τ_{b_1}) is a bms $T_{2\frac{1}{2}}$- space, $(\{p_1/x_1, q_1/y_1\}) \subseteq E_1$, $(\{p_2/x_2, q_2/y_2\}) \subseteq F_1$, and $\overline{E_1} \cap \overline{F_1} = \emptyset$. Thus, $(E_1, F_1) \in \tau_{b_2}$ and $(\{p_1/x_1, q_1/y_1\}) \subseteq E_1$, $(\{p_2/x_2, q_2/y_2\}) \subseteq F_1$, and $\overline{E_1} \cap \overline{F_1} = \emptyset$ in τ_{b_2}. Hence, (N_1, N_2, τ_{b_2}) is a bms $T_{2\frac{1}{2}}$- space.

4. Comparison of Relation Between Separation Axioms in Binary Multiset

In this section, the relation of T_i (i=0,1,2,3,4, and 5) axioms and their properties comparison of bms topological space are discussed.

Theorem 4.1. *If* (M_1, M_2, τ_{b_1}) *is bms* T_0*-space and* $\tau_{b_1} \leq \tau_{b_2}$*, Then,* (N_1, N_2, τ_{b_2}) *is also bms*T_0*-space.*

Proof. Let (M_1, M_2, τ_{b_1}) be a bms-T_0-space and $(\{p_1/x_1, q_1/y_1\}) \in M_1$, $(\{p_2/x_2, q_2/y_2\}) \in M_2$, there exist disjoint open bms $(\{P_X\}) \in \tau_{b_1}(\{p_1/x_1, q_1/y_1\})$ and $(\{Q_Y\}) \in \tau_{b_2}(\{p_2/x_2, q_2/y_2\})$ such that $(\{P_X\}) \cap (\{Q_Y\}) = \emptyset$. Assume $\tau_{b_1} \leq \tau_{b_2}$, which implies that every open bms in τ_{b_1} is also an open bms in τ_{b_2}. Therefore (N_1, N_2, τ_{b_2}) is a bms T_0-space, $(\{p_1/x_1, q_1/y_1\}) \in N_1$ and $(\{p_2/x_2,\ q_2/y_2\}) \in N_2$. Since, (M_1, M_2, τ_{b_1}) is a bms T_0-space, there exist open bms $(\{P/X\}) \in (\{p_1/x_1, q_1/y_1\})$ and $(\{Q/Y\}) \in (\{p_2/x_2, q_2/y_2\})$ such that $(\{P/X\}) \cap (\{Q/Y\}) = \emptyset$. Therefore, $(\{P/X\}) \in (\{p_1/x_1, q_1/y_1\})$ and $(\{Q/Y\}) \in (\{p_2/x_2, q_2/y_2\})$. Because $(\{P/X\}) \cap (\{Q/Y\}) = \emptyset$ in τ_{b_1}, it must also hold in τ_{b_2}. Hence, for any $(\{p_1/x_1, q_1/y_1\}) \in N_1$ and $(\{p_2/x_2, q_2/y_2\}) \in N_2$, are disjoint open bms $(\{P/X\}) \in (\{p_1/x_1, q_1/y_1\})$ and $(\{Q/Y\}) \in (\{p_2/x_2, q_2/y_2\})$. Hence (N_1, N_2, τ_{b_2}) is a bms T_0-space.

Theorem 4.2. *Every bms* T_1*-space is bms* T_0*-space.*

Remark 4.3. *Every discrete bms-topology* $(M_1, M_2, P^*(M_1, M_2))$ *is bms* T_1*-space. But, if* (M_1, M_2) *is a finite bms and* (M_1, M_2, τ_b) *is bms* T_1*-space not equal to the* $\tau_b = P^*(M_1, M_2)$ *in discrete bms-topology. As shown in the below example.*

Example 4.4. Let $M_1 = \{3/c, 1/d\}, M_2 = \{2/v, 5/u\}, \tau_b = \{(\emptyset, \emptyset), (M_1, M_2), (\{1/c, 1/d\}, \{2/v, 2/u\}), (\{2/c\}, \{3/u\}), (\{1/d\}, \{1/u\})\} \neq P^*(M_1, M_2)$. But (M_1, M_2, τ_b) is bms T_1-space.

Theorem 4.5. *If* (M_1, M_2, τ_{b_1}) *is bms* T_1*-space and* $\tau_{b_1} \leq \tau_{b_2}$*, Then,* (N_1, N_2, τ_{b_2}) *is also bms* T_1*-space.*

Proof. Let (M_1, M_2, τ_{b_1}) be a bms-T_0-space and for any $(\{p_1/x_1, q_1/y_1\}) \in M_1$ and $(\{p_2/x_2, q_2/y_2\}) \in M_2$, there exist disjoint open bms $(\{P/X\}) \in (\{p_1/x_1, q_1/y_1\})$ and $(\{Q/Y\}) \in (\{p_2/x_2, q_2/y_2\})$ such that $(\{P/X\}) \cap (\{Q/Y\}) = \emptyset$. Therefore every open bms in τ_{b_1} is also an open bms in τ_{b_2} and (N_1, N_2, τ_{b_2}) is a bms T_1-space in $(\{p_1/x_1, q_1/y_1\}) \in N_1$, $(\{p_2/x_2, q_2/y_2\}) \in N_2$. Since, (M_1, M_2, τ_{b_1}) is a bms-T_1-space, there exist open bms $(\{P/X\}) \in (\{p_1/x_1, q_1/y_1\})$ and $(\{Q/Y\}) \in (\{p_2/x_2, q_2/y_2\})$ $(\{p_1/x_1, q_1/y_1\}) \in N_1$ and $(\{p_2/x_2, q_2/y_2\}) \in N_2$, are disjoint open bms. Hence (N_1, N_2, τ_{b_2}) is a bms T_1-space.

Theorem 4.6. *The property of being bms* T_2*- space is hereditary property.*

Proof. Let (M_1, M_2, τ_b) be a bms T_2-space $(N_1, N_2) \subseteq (M_1, M_2)$, $(\{p_1/x_1, q_1/y_1\})$ and $(\{p_2/x_2, q_2/y_2\}) \subseteq (M_1, M_2)$, such that $(\{p_1/x_1, q_1/y_1\}) \neq (\{p_2/x_2, q_2/y_2\})$. Since, (M_1, M_2, τ_b) is bms T_2- space. Then, $(E, F) \in \tau_b$, $(\{p_1/x_1, q_1/y_1\}) \subseteq F$, $(\{p_2/x_2, q_2/y_2\}) \subseteq E$ and $E \cap F = \emptyset$. By the definition 3.8. of $(\{P/X\}) \cap E, (\{Q/Y\}) \cap F \in \tau_b$. Therefore $(\{p_1/x_1, q_1/y_1\}) \subseteq (\{P/X\}) \cap E$ and $(\{p_2/x_2, q_2/y_2\}) \subseteq (\{Q/Y\}) \cap F$. Since, $E \cap F = \emptyset$. Also, $((\{P/X\}) \cap E) \cap ((\{P/X\}, \{Q/Y\}) \cap F) = (E \cap F) \cap (\{P/X\}, \{Q/Y\}) = \emptyset \cap (\{P/X\}, \{Q/Y\}) = \emptyset$. Hence, (N_1, N_2) is bms T_2-space.

Theorem 4.7. *If (M_1, M_2, τ_{b_1}) is bms T_2-space and $\tau_{b_1} \leq \tau_{b_2}$, Then, (N_1, N_2, τ_{b_2}) is also bms T_2-space.*

Proof. The above condition obviously true for the Definition of 3.8 and Theorem 3.11.

Corollary 4.8. *Every bms sub-bms space of bms T_3-space is also a bms T_3-space.*

Theorem 4.9. *Every bms T_3-space is a bms-regular space.*

Proof. By using the Definition 3.13, and definition 3.14.

Remark 4.10. *The converse of the Theorem 3.15 is not true in general as following the example.*

Example 4.11. Let $M_1 = \{1/e, 3/f\}, M_2 = \{4/s, 5/r\}$ and $\tau_b = \{(\emptyset, \emptyset), (M_1, M_2), (\{1/s\}, \{2/f\}), (\{1/f, 1/e\}, \{2/s, 1/r\})\}$. Then, $\tau_b^c = \{(M_1, M_2), (\emptyset, \emptyset), (\{3/s, 5/r\}, \{1/e, 1/f\}), (\{2/f\}, \{2/s, 4/r\})\}$. Hence (M_1, M_2, τ_b) is a bms-regular space but not a bms-T_1- space.

Corollary 4.12. *The property of being bms T_4- space is bms topological property.*

Theorem 4.13. *The property of being bms -completely normal space is a hereditary property.*

Definition 4.14. *A bms-topological space (M_1, M_2, τ_b) is said to be bms T_5-space if:*

1. *(M_1, M_2, τ_b) is bms-completely normal space.*
2. *(M_1, M_2, τ_b) is bms T_1-space.*

Theorem 4.15. *Every bms T_5-space is a bms T_4-space.*

Theorem 4.16. *The property of being a bms T_5-space is a hereditary property.*

Theorem 4.17. *The property of being bms $T_{2\frac{1}{2}}$- space is a hereditary property.*

Proof. Let (M_1, M_2, τ_b) be a bms $T_{2\frac{1}{2}}$-space, $(N_1, N_2) \subseteq (M_1, M_2)$ and $(N_1,$

$N_2, \tau_b)$ be a bms subspace of (M_1, M_2, τ_b). Therefore, (N_1, N_2, τ_b) is bms $T_{2\frac{1}{2}}$-space, $(\{p_1/x_1, q_1/y_1\}), (\{q_1/y_1, q_2, y_2\}) \subseteq (M_1, M_2)$, $(\{p_1/x_1, q_1/y_1\}) \neq (\{p_2/x_2, q_2/y_2\})$. There exist $(E, F) \in \tau_b$ such that $(\{p_1/x_1, q_1/y_1\}) \subseteq E, (\{q_1/y_1,\ q_2, y_2\}) \subseteq F$, and $\overline{E} \cap \overline{F} = \emptyset$. By the definition 3.23 of sub-bms $(M_1 \cap E), (M_2 \cap F) \in \tau_b$. Therefore $\{p_1/x_1, p_2/x_2\} \subseteq (M_1 \cap E), (\{q_1/y_1, q_2, y_2\}) \subseteq (M_2 \cap F)$. Also, $\overline{(M_1 \cap E)} \cap \overline{(M_2 \cap F)} \subseteq (\overline{E} \cap \overline{M_1}) \cap (\overline{F} \cap \overline{M_2}) = (\overline{E} \cap \overline{F}) \cap \overline{(M_1, M_2)} = \emptyset \cap \overline{(M_1, M_2)} = \emptyset$, $\overline{(M_1 \cap E)} \cap \overline{(M_2 \cap F)} = \emptyset$. Hence (M_1, M_2, τ_{b_1}) is bms $T_{2\frac{1}{2}}$-space.

Theorem 4.18. *Every bms $T_{2\frac{1}{2}}$- space is a bms T_2-space.*

Proof. Let (M_1, M_2, τ_b) be a bms $T_{2\frac{1}{2}}$- space, by definition 3.23., $(\{p_1/x_1, q_1/y_1\})$ and $(\{p_2/x_2, q_2/y_2\})$ in (M_1, M_2) and $(\{p_1/x_1, q_1/y_1\}) \neq (\{p_2/x_2, q_2/y_2\})$, $(E, F) \in \tau_b$ such that $(\{p_1/x_1, q_1/y_1\}) \subseteq E$, $(\{p_2/x_2, q_2/y_2\}) \subseteq F$, $\overline{E} \cap \overline{F} = \emptyset$. Therefore, (M_1, M_2, τ_b) is a bms T_2-space, for any two distinct bms points $(\{p_1/x_1, q_1/y_1\}) \neq (\{q_/y_1, q_2y_2\})$ in (M_1, M_2), and $(\{p_1/x_1, q_1/y_1\}) \in M_1$ and $(\{q_1/y_1, q_2/y_2\}) \in M_2$. Therefore $(\{p_1/x_1, q_1/y_1\}) \neq (\{p_2/x_2, q_2/y_2\})$, by the definition 3.23., of a bms $T_{2\frac{1}{2}}$- space, there exist open bms E and F in τ_b such that $(\{p_1/x_1, q_1/y_1\}) \subseteq E$, $(\{p_2/x_2, q_2/y_2\}) \subseteq F$, and $\overline{E} \cap \overline{F} = \emptyset$. Hence, every bms $T_{2\frac{1}{2}}$- space is a bms T_2-space.

5. Characterization of Separation Axioms in Binary Multiset

In this section, the concept of bms separation axioms T_0, T_1, T_2, T_3, T_4 and T_5 spaces and explore some of their characterizations are derived.

Theorem 5.1. *Let (M_1, M_2, τ_b) be a bms topological space for (i) Every bms T_0-space is bms T_0-space.*
(ii) Every bms T_1-space is bms T_1 -space.
(iii) Every bms T_2 -space is bms T_2 -space.
(iv) Every bms T_1- space is bms T_0-space.
(v) Every bms T_2 -space is bms T_0-space.
(vi) Every bms T_2 -space is bms T_1 -space.

Proof. (i) Let (M_1, M_2) be a bms T_0-space $(\{p_1/x_1, q_1/y_1\})$ and $(\{p_2/x_2, q_2/y_2\})$ a two distinct points of (M_1, M_2) is a bms T_0-space, there exist open bms (M_1, M_2). Such that $(\{p_1/x_1, q_1/y_1\}) \in M_1$ and $(\{p_2/x_2, q_2/y_2\}) \in M_2$. There exist open bms (M_1, M_2) such that $(\{p_1/x_1, q_1/y_1\}) \in M_1$ and $(\{p_2/x_2, q_2/y_2\}) \in M_2$ changes occurred in $(M_1, M_2)$is bms T_0-space.
(ii) Proof of (ii) to (vi) is obvious.

Theorem 5.2. *In bms topological space, the following*
(i) Every bms T_4-space is a bms T_3-space.

(ii) Every bms-normal space is not necessarily bms-regular.

Proof. (i) Let (M_1, M_2, τ_b) be a bms T_4-space so that by definition 3.20 it is a bms T_1-space as well as bms-normal space, and bms T_1-space to be bms T_3 as known as bms-regular. Let (M_1, M_2) be any sub-bms of $(A, B), (C, D)$ and $(\{p_1/x_1, q_1/y_1\}) \in (A, B)$ and $(\{p_2/x_2, q_2/y_2\}) \in (C, D)$. Hence (M_1, M_2) in sub-bms of bms T_1-space.

Remark 5.3. *The following diagram is relation on the separation axioms in bms topological space is reversible process not in irreversible.*

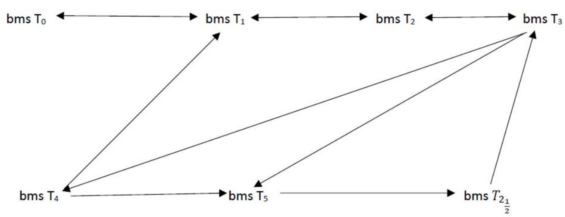

Figure :1

6. Application of Separation Axioms in Binary Multiset

A gene mutation is a permanent alteration in the DNA sequence that makes up a gene. These changes can occur naturally during DNA replication or be triggered by external factors like radiation, chemicals, or viruses. Mutations can affect how proteins are made, potentially altering traits or causing genetic disorders. While many mutations are harmful, some can be beneficial or neutral—and they're a key driver of evolution. DNA sequencing involves reading the order of nucleotides (A, T, C, G) in a DNA molecule A bms can model DNA sequences when repeated elements and binary relationships of (e.g. base pairing) are crucial.

Example 6.1. Let $\mathbb{M}_{5'}^{3'} = GGATCC = \{2/G, 1/A, 1/T, 2/C\}, \mathbb{M}_{3'}^{5'} = CCTAGG = \{2/C, 2/G, 1/A, 1/T\}$, be genetic sequence of binary multiset, $\tau_b = \{(\phi, \phi), (\mathbb{M}_{5'}^{3'}, \mathbb{M}_{3'}^{5'}), (\{1/T, 1/A\}, \{1/C, 1/G\}), (\{2/C\}, \mathbb{M}_{5'}^{3'}), (\mathbb{M}_{3'}^{5'}, \{1/G, 1/A\})\}$. Since $(U, V) \subseteq (\{1/T, 1/A\}, \{1/C, 1/G\}) = \tau_b$ is a bms T_0- space.

Example 6.2. Let $\mathbb{M}_{5'}^{3'} = GGATCC = \{2/G, 1/A, 1/T, 2/C\}, \mathbb{M}_{3'}^{5'} = CCTAGG = \{2/C, 2/G, 1/A, 1/T\}$, be genetic binary multiset $\tau_b = \{(\phi, \phi), (\mathbb{M}_{5'}^{3'}, \mathbb{M}_{3'}^{5'}), (\{1/T, 1/A\}, \{1/C, 1/G\}), (\{2/C\}, \mathbb{M}_{5'}^{3'}), (\mathbb{M}_{3'}^{5'}, \{1/G, 1/A\})\}$. Thus, $(\{1/G, 1/A\}) \subseteq M_1$, $(\{1/C, 1/G\}) \subseteq M_2$ its bms T_0-space but not bms T_1-space.

Example 6.3. Let $\mathbb{M}_{5'}^{3'} = ACTAG = \{2/A, 1/C, 1/T, 1/G\}, \mathbb{M}_{3'}^{5'} = CTAGA = \{1/C, 1/T, 2/A, 1/G\}$ be a genetic bms topological space $\tau_b = \{(\phi, \phi), (\mathbb{M}_{5'}^{3'}, \mathbb{M}_{3'}^{5'}), (\{1/A\}, \{1/T\}), (\{1/C\}, \{1/G\}), (\{1/G, \}, \{1/C\}), (\{1/T\}, \{1/A\})\}$. Its a bms T_3

-space. Here M and N are different by the τ_b is mutation. Because the parining of genetic code does not equal the amino acid so the starts of mutation.

Example 6.4. Let M=GGGCAGUCUC CCGGCGUUUA AGGGAUCCUG AACUUCGUCG= $\{13/, 21/C, 6/A, 10/U\}$, N=CUCCCAUCCA AUCAGUCCGC CUCACGGAUG GAGUUG =$\{8/G, 13/C, 8/U, 6/A\}$ be a genetic coding $\tau_b = \{(\phi, \phi), (M, N), (\{2/G\}, \{2/G\}), (\{10/G\}, \{6/C\}), (\{6/A\}, \{6/U\}), (\{3/U\}, \{2/A\}), (\{15/G\}, \{10/C\})\}$. Its satisfied bms T_4-space, and $M \neq N$ is formed nonsense mutation. A genetic change that introduces a premature stop codon in DNA sequence. This often leads to a truncated and non-functional protein.

7. Conclusion

In this paper, introduce the separation axiom in binary multiset topological space. The behavior of these axioms under various types of mappings is examined.The separation axioms $T_0, T_1, T_2, T_3, T_4, T_5$ and $T_{2\frac{1}{2}}$ are fundamental concepts in bms topology.These axioms provide a hierarchy of bms topological properties that are crucial in understanding the structure and properties of binary multiset topological spaces. Also using some application of DNA sequence problem are discussed.

References

[1] Abed A. Khrija, Binary D-Separation Axioms in Binary Topological Space, Galoitica: Journal of Mathematical Structures and Applications, 9(2), (2023), 49-55.

[2] El-Atik A. A., Tashkandy Y., Jafari S., Nasel A. A., Eman W. and Badr M., Mutation of DNA and RNA sequence through the application of topological spaces, AIMS Mathematics, 8(8), (2023), 19275-19296.

[3] El-Sharkasy M. M. and Sundes Rajeed Badr, Multiset topology via DNA and RNA mutation, Mathematical meethods in the applied science, (2017), 1-18.

[4] El-Sheikh Sobhy, Omar Rajab, and Mahmoud Raafat, Separation Axioms on Multiset Topological Space, Journal of New Theory, 7, (2015), 11-21.

[5] Girish K. P., John Jacob Sunil, On Multiset Topologies, Theory and Applications of Mathematics & Computer Science., 2 (2012b), 37-52.

[6] Hoque M. M., Bhattacharya B. and Tripathy B. C., New Separation Axioms in Multiset Topology, Bol. Soc. Paran. Mat., 2024(42), (2023), 1-10.

[7] Nithyanantha Jothi S., Thangavelu P., On binary continuity and separation axioms, Ultra Sci. 24 (2012), 121-126.

[8] Nithyanantha Jothi S., Contribution to Binary Topological Spaces, Ph.D. Thesis, Manonmaniam Sundaranar University, Tirunelveli, 2012.

[9] Nithyanantha Jothi S., Thangavelu P., On Binary Topological Spaces, Pacific. Asian. Journ. Math, 5(2), (2011), 95-107.

[10] Priyalatha S. P. R., Sowndariya R., Saleh S., Al-Mufarrij J., Binary Multiset topological spaces: A new structure, Mathematical Sciences Letters An international Journals, 3 (2023), 75-83.

[11] Surekha S. S., Sindhu G., New class of open sets in binary topological space, International journals of creative research thoughts (IJCRT), 10(5), (2022), 101-104.

[12] Shravan K. and Tripathy B. C., Metrizability of multiset topological spaces, Bulletin of the Transilvania University of Brasov, Series III: Mathematics, Informatics, Physics, 13(62), 2 (2020), 683-696.

[13] Tong J., A separation axiom between T_0 and T_1, Ann. Soc. Sci., 96 (1982), 85-90.